An implementation of linear system solving for integer matrices

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We present a fast implementation of the well-known p-adic lifting algorithm for computing an exact solution vector to a system of linear equations with integer coefficients. A feature of this problem is that the solution typically contains large, multi-precision rational numbers even if the input system contains word-size integers. By combining a careful choice of the lifting modulus with a residue number system the lions share of computation is done in 53-bit precision. This allows computing all matrix-matrix and matrix-vector products using the portable and highly optimized numerical BLAS library. The remaining computation with multi-precision numbers use the GMP bignum library. On a modern workstation the system solver will compute the exact solution to a system of dimension two thousand with single digit entries in under two minutes. The numerators and denominators of entries in the solution vector have over four thousand decimal digits.