A Matlab package computing polynomial roots and multiplicities

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Software name: MultRoot

Short description: MultRoot is a numerical root-finder for general polynomials with following features:

- High accuracy in computed roots, especially multiple roots.
- Accurate computation of root multiplicities.
- Automatic verification of the results with the backward error, the estimated forward error, as well as the pejorative condition number.
- Permits inexact input coefficients.
- Uses standard machine precision without extension.
- An easy-to-use black-box software. The only required input is the coefficient vector.

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Abstract

MultRoot is a collection of Matlab modules for accurate computation of polynomial roots, especially roots with high multiplicities, using standard machine precision. As a blackbox-type software, MultRoot requires the polynomial coefficients as the only input, and outputs the computed roots, multiplicities, backward error, estimated forward error, as well as the pejorative condition number.

There are two common limitations for standard numerical root-finding software when multiple roots are present. Namely, those methods suffer from severe loss of accuracy and lack the capacity of multiplicity identification. Symbolic polynomial factorization requires *exact* rational coefficients. In contrast, the most significant features of MultRoot are the multiplicity identification capability and the remarkable accuracy on multiple roots without using the multiprecision arithmetic, even if the polynomial coefficients are inexact.

There is a so-called "attainable accuracy" for conventional root-finders: the attainable number of corrected digits of a computed root is limited by the data/machine precision divided by the multiplicity. For roots with high multiplicities, this accuracy barrier suggests that multiprecision arithmetic is required in addition to exact coefficients. Using a novel approach based on the pejorative manifold theory, MultRoot achieves high accuracy even if the polynomial is perturbed and the machine precision is not extended. A stable numerical polynomial GCD-finder is also developed as an essential component that determines the multiplicity structure.

MultRoot is a combination of two programs that can be used independently. GcdRoot calculates the multiplicity structure and initial root approximation. PejRoot refines the root approximations by projecting the polynomial onto a prescribed pejorative manifold. A comprehensive test suit of polynomials that are collected from the literature is included for numerical experiments and performance comparison.

The detailed algorithm and analysis is presented in this conference [1].

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Example: For the polynomial

$$p(x) = (x-1)^{20}(x-2)^{15}(x-3)^{10}(x-4)^{5}$$

in general form, with coefficients truncated at 16-th digits. The MultRoot, using standard machine precision, outputs results:

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>> multroot(p)

      THE PEJORATIVE CONDITION NUMBER:                76.7076
      THE BACKWARD ERROR:                    6.16e-016
      THE ESTIMATED FORWARD ROOT ERROR:      9.46e-014

         computed roots         multiplicities

         3.999999999999985            5
         3.000000000000011           10
         1.999999999999997           15
         1.000000000000000           20
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References

[1] Z. Zeng, *A method computing multiple roots of inexact polynomials*, Proceedings of ISSAC 2003.